Lesson 9.7 Equations of motion

Recommended teaching time for this lesson: 1.5 x 60 minute periods

Period 1

• 35 minutes of explicit teaching

• 25 minutes of suggested classroom activities

• 30 minutes homework

Period 2

• 15 minutes of explicit teaching

• 15 minutes of suggested classroom activities

• 30 minutes homework

Getting started

Key ideas

* The definitions of velocity and acceleration can be turned into formulas for solving problems algebraically.
* These formulas can be rearranged and combined to produce new formulas.
* For uniform acceleration of an object from rest, the final velocity is twice the average velocity.
* Negative acceleration can mean slowing down or speeding up, depending on the direction of travel.

Curriculum links

Science understanding

* Solve problems relating to uniformly accelerated motion in one dimension using .

Advice for teaching this lesson

Things to know before you start teaching

The syllabus presents only three of the potential five *suvat* equations; however, four are given in a summary table in this chapter, with the fifth being broken into two halves. While they will use these equations in Unit 3 in a slightly different form, the two extra *suvat* equations are not presented at all. This does not mean that they are not valid to use; however, students should be cautioned that using an incorrectly remembered equation could potentially result in more marks lost.

Simultaneous equations for dual moving systems will reappear on the harder problems in this lesson and it is important that students recall that mathematical technique.

It is recommended that you use some of the 1.5 hours assigned to complete the ‘Analytical processes’ section of the Check your learning activity in class to ensure students are able to tackle the increasing difficulty of questions.

Common misconceptions

* For scenarios with two segments – like a car accelerating to a constant speed – students often fail to understand that the final velocity of the first time segment can become the initial velocity of the second time segment. Check for understanding on this.

Differentiation strategies

Encourage students to create summary tables of given and desired variables when solving questions. Requiring students to identify and list out all *suvat* variables can often overcome the first problem of reading a question and placing it into abstract mathematical terms.

Starter activity: Independence of vectors

Approximate time: 5 minutes

**Activity placement:** Place directly after Lesson overview

**Activity summary:** Evaluating a claim about velocity and acceleration.

Notes for the teacher

Students should arrive at an answer of ‘yes, it is possible’.

This could be conducted as a class discussion/debate if preferred.

Instructions for students

Read the claim below.

“Velocity can be positive at the same time as acceleration is negative.”

* 1. Is this statement true or false? Justify your answer.

Answers

The statement is true. Velocity and acceleration are different properties. While acceleration is the change of velocity, you can have a positive velocity (such as moving forwards) with a negative acceleration (slowing down to a stop) such as at the end of a running race, or a car approaching a stop sign. In both these situations the direction of the acceleration and velocity will be opposite.

Classroom activity: Why does that work?

Approximate time: 10 minutes

**Activity placement:** Place directly above “Skill drill”

**Activity summary:** An activity to demonstrate there are multiple ways to use equations.

Notes for the teacher

This activity is more of a mathematical reasoning activity; however, the introductory concept can benefit students who dislike using the quadratic equation.

Students should recognise that the first equation is in the form of a quadratic and hence the quadratic equation from their math studies can be used to solve for ‘t’ when needed.

Instructions for students

Step 1: In physics we solve questions that involve determining an unknown value. In the equations you just saw, each variable is only used once, except in the formula: . This formula is a quadratic that you may remember from year 10 or earlier mathematics, and as such can be solved using the quadratic equation: .

1. Rewrite the quadratic equation using the symbols from the *suvat* formula to solve for ‘t’. Be careful with replacing the ‘*a*’*s*’.

Step 2: However, there is another way to solve this problem if you are given *s*, *u*, and *a*. If instead you use to solve for ‘v’, you can then use to solve for ‘t' and still obtain the same answers. Note that the square root for ‘v’ will create two potential answers that can be used.

1. Compare the two *suvat* equations given in Step 2 with the quadratic equation you substituted in question ‘a’ and identify the identical components.

Helpful hints

* Consider what part you would do first on the quadratic equation. Remember your order of operations.

Answers

1. Note that the a, b, c for lines 3 to 5 on the left refer to the general form of the quadratic equation, while the right hand side refer to variables from the *suvat* equation.
2. The is the solution for v in . The is the solution for t in . The ± is needed as there are two solutions for ‘v’ due to being solved by a square root.

Classroom activity: Negative acceleration and changing direction

Approximate time: 10 minutes

**Activity placement:** Place directly above “Real-world physics”

**Activity summary:** A problem designed to have students consider ‘which’ solution is correct.

Notes for the teacher

If students do the mathematics of this correctly they will obtain two possible answers.

Encourage students to consider the narrative of what would happen, and where their solutions would happen.

Instructions for students

Step 1: Solve the following question.

1. A person shoots a ball towards a basketball hoop. The hoop is 1.5 metres above their hands when the ball is released. They shoot with an upwards velocity of 6 m/s, while the ball accelerates downwards with a magnitude of 9.8 m/s2. Ignore the horizontal motion of the basketball. Calculate how much time it would take the ball to reach the same height as the hoop.
2. Identify what is happening for each solution.

Helpful hints

* Consider in which direction the acceleration acts compared to the initial velocity of the ball.

Answers

1. t = 0.35 s or t = 0.87 s
2. The first time is when the ball goes up past the hoop, and then second time is when the ball comes back to the same height as the hoop.

Classroom activity: Simultaneous systems

Approximate time: 15 minutes

**Activity placement:** Place directly above “Check your learning 9.7”

**Activity summary:** A question that introduces students to simultaneous equations for systems of motion and demonstrates how to solve them.

Notes for the teacher

The support level could be used as a valid teaching tool for the entire class before assigning out normal/challenges.

This activity, even at the normal level, will be of great use to your students aiming for the highest marks as this is a common ‘trick’ used in physics problems.

Instructions for students

Step 1: Read the scenario below and answer the following questions.

“A car driving at a constant 20 m/s passes a stationary police car. The police car accelerates at a rate of 1.2 m/s2 and catches the speeding driver down the street.”

1. Identify how many moving systems you have in this scenario.
2. List all known and unknown *suvat* variables for the question.

Step 2: As you can see from listing out the variables, you have too many unknown variables to solve for just one system. For a constant velocity system you need two known and one unknown variable, while for an accelerating system you need at least three known variables to solve for two unknown variables.

1. Write two equations that involve displacement and time. One for the speeding car, and one for the police car.

Step 3: Because both systems ‘meet’, this means at this point they will have the same time and displacement. As such we can let the two equations equal each other.

1. Solve for the time it takes the police car to catch the speeding car.
2. Solve for the distance it takes the police car to catch the speeding car.
3. Create a graph with both displacement−time functions on the same set of axes.

Helpful hints

* Use the equations you created in part c to generate a set of ‘s’ values for different ‘t’s.

Support activity

Notes for the teacher

This removes the graphing activity and demonstrates some of the steps that students need to take with more direction.

Instructions for students

Step 1: Read the scenario below and answer the following questions.

“A car driving at a constant 20 m/s passes a stationary police car. The police car accelerates at a rate of 1.2 m/s2 and catches the speeding driver down the street.”

1. The speeding car is one moving system in this question. Identify the other moving system.
2. List all known and unknown *suvat* variables for the two systems. Make sure to separate them into two columns to keep your information clear.

Step 2: As you can see from listing out the variables, you have too many unknown variables to solve for just one system. For a constant velocity system you need two known and one unknown variable, while for an accelerating system you need at least three known variables to solve for two unknown variables.

1. Write two equations that involve displacement and time. One for the speeding car, and one for the police car.

Step 3: Because both systems ‘meet’, this means at this point they will have the same time and displacement. As such we can let the two equations equal each other. Do this by getting displacement on its own in both of your equations.

1. Solve for the time it takes the police car to catch the speeding car.
2. Solve for the distance it takes the police car to catch the speeding car.

Challenge activity

Notes for the teacher

This creates a different challenge problem for students to solve after doing the normal version.

Instructions for students

Step 1: Read the scenario below and answer the following questions.

“A car driving at a constant 20 m/s passes a stationary police car. The police car accelerates at a rate of 1.2 m/s2 and catches the speeding driver down the street.”

1. Identify how many moving systems you have in this scenario.
2. List all known and unknown *suvat* variables for the question.

Step 2: As you can see from listing out the variables, you have too many unknown variables to solve for just one system. For a constant velocity system you need two known and one unknown variable, while for an accelerating system you need at least three known variables to solve for two unknown variables.

1. Write two equations that involve displacement and time. One for the speeding car, and one for the police car.

Step 3: Because both systems ‘meet’ this means at this point they will have the same time and displacement. As such we can let the two equations equal each other.

1. Solve for the time it takes the police car to catch the speeding car.
2. Solve for the distance it takes the police car to catch the speeding car.
3. Create a graph with both displacement-time functions on the same set of axes.
4. Repeat question ‘d’ but instead of the police car having limitless acceleration, limit their final velocity to 25 m/s. Hint: You’ll need to figure out how far each travels as the police car speeds up first.

Answers

1. 2 systems
2. Car Police  
   s = ? s = ?  
    t = ? t = ?  
    v = 20 v = ?  
    u = 0  
    a = 1.2
3. Car:   
   Police:
4. Link the two equations from Step c.
5. Solve with either equation from part c with solution from part d.

Support activity

1. Police car
2. Car Police  
   s = ? s = ?  
   t = ? t = ?  
   v = 20 v = ?  
    u = 0  
    a = 1.2
3. Car:   
   Police:
4. Link the two equations from Step c.
5. Solve with either equation from Step c with solution from Step d.

Challenge activity

1. 2 systems
2. Car Police  
   s = ? s = ?  
   t = ? t = ?  
   v = 20 v = ?  
    u = 0  
    a = 1.2
3. Car:   
   Police:
4. Link the two equations from Step c.
5. Solve with either equation from Step c with solution from Step d.
7. Working out time to maximum speed of police car:  
   Working out distance both cars travel during this time and the distance between them:  
   Working out the time it would take the police car to catch up the 156 m gap.